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1999 J. Phys.: Condens. Matter 11 7723

(http://iopscience.iop.org/0953-8984/11/40/305)

J. Phys.: Condens. Matter **11** (1999) 7723–7736. Printed in the UK PII: S0953-8984(99)99889-2

Finite-wavevector studies of two-dimensional systems

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Received 3 December 1998

Abstract. The results of ballistic phonon and microwave absorption experiments are presented in the fractional quantum Hall regime of two-dimensional electron and hole systems respectively. Time-resolved ballistic phonon results suggest that acoustic phonons can be absorbed by the magnetoroton excitations of a fractional quantum Hall fluid. The technique allows the determination of the magnetoroton gap and holds out the promise of being able to measure the magnetoroton dispersion curve in angle-resolved experiments. At lower Landau level filling factors, high-mobility two-dimensional systems become insulating. Microwave absorption experiments at finite wavevector find a series of sharp absorption lines in the insulating phase that become more pronounced as the temperature is lowered and the magnetic field increased. The results strongly suggest that the two-dimensional system forms a pinned Wigner solid at these low filling factors. Further analysis allows the determination of the pinning frequency and its variation as a function of magnetic field.

1. Introduction

My interest in two-dimensional carrier systems (2DCS) began when I was a research student working on ions trapped beneath the surface of superfluid helium. It was a privilege to have Professor W F Vinen as my supervisor. His guidance and support throughout my time in Birmingham were invaluable. As a post-doctoral researcher, again working with Professor Vinen, I studied Wigner crystallization in the ion system. Since moving to Nottingham I have continued to study low-dimensional systems, this time in semiconductors using finite-wavevector techniques to probe the fractional quantum Hall effect (FQHE) and magnetically induced Wigner solidification (MIWS). In this article, I would like to summarize some of our research in Nottingham and show how ideas that were developed to explain excitations in superfluid helium and Wigner crystallization of electrons and ions at the surface of superfluid helium, both fields in which Professor Vinen has made major contributions, have been used as a starting point to develop theoretical models of the phenomena seen in GaAs/AlGaAs heterostructures.

The properties of high-mobility 2DCS have mainly been studied in two semiconductor structures. The first to be exploited was the silicon MOSFET in which a 2D electron system (2DES) forms at the interface between the silicon surface and an oxide layer. The carriers are accumulated at the interface by applying a voltage to a metal gate fabricated on top of the oxide layer. It was in this system that von Klitzing, Dorda and Pepper [1] discovered the integer quantum Hall effect in magnetotransport measurements made with a magnetic field applied perpendicular to the sheet of charge. Von Klitzing was awarded the Nobel Prize in 1985 for this work.

The application of a perpendicular magnetic field to the 2DCS causes the formation of Landau levels in which the density of states in a disorder-free sample at zero temperature becomes a series of δ-functions at energies $E = \hbar\omega_c(n + 1/2)$ where ω_c is the cyclotron frequency of the carriers and n is an integer. Each Landau level has a degeneracy of eB/h states per unit area. The presence of disorder in the sample broadens each Landau level and alters the nature of the carrier states. The states that lie between the Landau levels are localized, whilst a narrow core of states within each Landau level are extended. Only extended states can carry current at zero temperature. The magnetotransport properties of the system are determined by whether the Fermi level lies within a Landau level where scattering to nearby unoccupied states is possible or between the Landau levels where it is not. To discuss the results of von Klitzing's experiments it is useful to introduce the quantity known as the filling factor, ν, that defines how many Landau levels are occupied. When the Fermi level lies between Landau levels, a mobility gap opens up and the longitudinal magnetoresistance tends towards zero whilst the Hall resistance takes a quantized value, $R_H = h/e^2\nu$. The most significant thing about the values of R_H on the plateaus is that it depends only on fundamental constants and not the material parameters. The conditions necessary for the observation of this effect is that the mobility of the carriers is high and the temperature is low enough to ensure that thermal excitation from a filled Landau level to the Fermi level is highly unlikely and that the carriers are degenerate. Further discussion of the integer and fractional quantum Hall effects can be found in Prange and Girvin [2].

The integer effect may be explained in the framework of non-interacting electrons. In this framework, when the filling factor is less than one, all the carriers lie in the lowest Landau level and no further plateaus in the Hall resistance are expected. However, in 1982, Störmer, Tsui and Gossard [3] discovered further plateaus at $\nu < 2/3$ and 1/3, the first observation of the *fractional* quantum Hall effect (FQHE) in which R_H takes the quantized values h/fe^2 where $f = p/q$; p and q are integers with q odd. Horst Störmer, Daniel Tsui and the theorist Robert Laughlin have recently been awarded the 1998 Nobel Prize in Physics for the discovery of this novel quantum fluid.

The sample studied was a 2DES formed in a GaAs/AlGaAs heterojunction. In this device, a 2D sheet of carriers is trapped at the interface between GaAs and AlGaAs. The band gap of the AlGaAs is larger than that of GaAs. The AlGaAs is deliberately doped with impurities and the mobile carriers migrate to the interface. The attraction between the ionized dopants and the mobile carriers causes bending of the conduction and valence bands. The mobile carriers are trapped in the resulting triangular potential well forming a two-dimensional layer with a thickness, for a 2DES, of the order of 5 nm, for a typical carrier density of 10^{11} cm^{-2}. The separation of the dopants from the carriers leads to reduced carrier–dopant scattering and so a high mobility.

An alternative definition of the filling factor is that $\nu = N_e/N_\phi$, where N_e is the number of electrons in the sheet and N_ϕ is the number of magnetic flux quanta threading the system. Magnetic flux is quantized in units of h/e, where h is Planck's constant and e is the charge on an electron, so N_ϕ is given by the magnetic flux through the plane divided by h/e. Therefore, at $\nu = 1/3$, there are three flux quanta per carrier threading the system. In 1983, Laughlin suggested that the fractional quantum Hall effect, close to filling factors of the form $1/m$, where m is an odd integer, is due to the formation of a new type of highly correlated quantum fluid that is incompressible and possesses quasiparticle excitations with fractional charge [4]. He gave an explicit construction of the ground-state wavefunction for quantum liquid states which numerical calculations of the eigenstates of systems with small numbers of electrons subsequently showed to be excellent [5].

Girvin, MacDonald and Platzman (GMP) [6] used Laughlin's ground-state wavefunction to investigate the low-energy collective modes of the incompressible fluid using techniques developed by Feynman to study superfluid helium [7]. At first sight it is surprising that such a technique is applicable to a Fermi system. It is well known that Fermi systems show a high density of single-particle excitations in the form of particle–hole excitations across the Fermi surface [8]. However, in two dimensions the strong magnetic field destroys the Fermi surface. As already discussed, the single-particle quantum states are highly degenerate Landau levels separated by gaps in the kinetic energy of the order of $\hbar\omega_c$. Excitations involving the transfer of a carrier from one Landau level to the next will have an energy of this order. Intra-Landau level excitations will have much lower energies of the order of the Coulomb energy $e^2/4\pi\varepsilon_0\varepsilon l_B$ where l_B is the magnetic length. It is these latter excitations which are collective in nature that can be described by the GMP theory.

The basis of Feynman's theory is to describe the low-energy collective excitations of the quantum fluid in terms of the properties of the quantum ground state of the system. In the case of the FQHE, the quantum ground state is described very well by Laughlin's wavefunction. Feynman finds that the energy $\Delta(k)$ of the low-lying excitations is given by $\Delta(k) = f(k)/s(k)$, where $f(k)$ is the oscillator strength and $s(k)$ is the static structure factor, which for superfluid helium can be determined from neutron scattering experiments. In the fractional quantum Hall fluid, GMP find analogous expressions for $\Delta(k)$, $\Delta(k) = \bar{f}(k)/\bar{s}(k)$ where $\bar{f}(k)$ and $\bar{s}(k)$ are the oscillator strength and static structure factor respectively, projected onto the lowest Landau level. In this case, although it is impossible to measure $\bar{s}(k)$ in a direct experiment, it is possible to calculate it from Laughlin's wavefunction; $\bar{f}(k)$ can also be calculated. The result is that the dispersion curve for $\Delta(k)$ is finite at low wavevectors and has a pronounced minimum at a wavevector corresponding to the peak in the projected static structure. By analogy with superfluid ^{4}He, where a peak in $s(k)$ leads to the roton minimum, this minimum in the dispersion curve is known as the magnetoroton minimum. This theoretical model can only be used at fractions such as $\nu = 1/3$, $1/5$, etc as Laughlin's ground-state wavefunction is only defined at these filling factors. More recently, an alternative picture of the FQHE has been developed in which the quasiparticles are composite fermions consisting of a carrier bound to an even number of flux quanta [9]. The FQHE can then be described as the IQHE of the composite fermions. A clear introduction to the research in this area has been given by Nicholas [10]. Using this new theoretical framework, Kamilla *et al* [11] have calculated the dispersion curves of the collective excitations for other values of ν such as 2/5. The calculations confirm the theory of GMP at $\nu = 1/3$.

2. Phonon studies of the magnetoroton minimum

The roton minimum in superfluid helium has proved crucial to the understanding of the properties of the fluid. It is therefore of great interest to observe the magnetoroton minimum directly and investigate the interaction of these excitations with other quasiparticles such as the acoustic phonons of gallium arsenide. The two experimental approaches to this are resonant Raman scattering and acoustic phonon absorption. Resonant Raman scattering experiments [12] rely on disorder present in the sample to couple the photons to the large in-plane wavevectors of the magnetorotons. Through the use of low-density samples and tilting the incident laser light with respect to the plane of the carriers, in-plane wavevectors of around 0.15 ql have been reached [13]. To couple to higher wavevectors the experiments rely on the disorder to allow the light to couple to the magnetoroton minimum at $ql \sim 1.3$. This means however that the wavevector of the excitations is unknown. Excitations of the correct energy have only been seen for $\nu = 1/3$. Acoustic phonons have the potential to probe the magnetoroton minimum with a known range of in-plane wavevectors. A schematic diagram comparing the acoustic phonon dispersion to the magnetoroton dispersion is shown in figure 1.

In a series of experiments at Nottingham University we have developed techniques to allow us to observe time-resolved ballistic phonon absorption in the FQH regime. The time resolution is much faster than the typical phonon time of flight in our samples and we can resolve interactions with different phonon modes and angles of incidence. Whilst experiments have been carried out at several filling factors [14] we will concentrate here on experiments at $\nu = 1/3$.

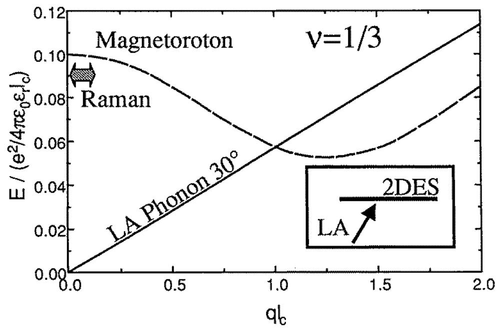

Figure 1. A schematic diagram of the MR dispersion curve and acoustic phonon dispersion curve showing that for typical angles of incidence creation of magnetorotons by the absorption of acoustic phonons is a likely interaction. Also shown on the graph are the range of known in-plane wavevectors so far reached by resonant Raman scattering.

The two samples used in these experiments are n-type heterojunctions grown on a 2 mm semi-insulating GaAs substrate. The properties of the two samples are summarized in table 1. Due to the extremely small effect of the phonon absorption on the resistance of the 2DES, it has been patterned into a meandering path that covers a 5×5 mm^2 area (sample A) or a 1×1 mm^2 area (sample B). The length-to-width ratio of the meander is $\sim$300 for both samples. To avoid the effects of contacts being heated by the phonon pulse the meander is positioned towards one end of the 11×11 mm^2 sample. The contacts are positioned in the two opposite corners of the sample. This allows the experimental readings to be completed before the ballistic phonons reach the contacts. On the rear surface facing the centre of the meander line, a thin-film constantan heater, also in the form of a 1×1 mm^2 meander, was fabricated. On sample B an additional heater was positioned such that the centre of the heater subtended an angle of 45° to the centre of the meander line. The majority of phonons incident on the 2DES from this heater propagate in the [110] plane; preliminary results of experiments with angular resolution are reported in [15].

Table 1. Densities and mobilities of samples A and B used in the phonon absorption experiments.

Sample	n_s (cm^{-2})	μ (cm^2 V^{-1} s^{-1})
A	1.16×10^{11}	1.5×10^6
B	1.21×10^{11}	1.2×10^6

The sample was mounted, in vacuo, on the tail of a dilution refrigerator. The meander line was electrically connected to a room temperature preamplifier using a low-capacitance coaxial cable. A pulse of non-equilibrium phonons was created by applying a short voltage pulse, typically of duration 20–50 ns, to the thin-film heater. The non-equilibrium temperature,

T_h, of phonons entering the cold substrate can be calculated from the power dissipated in the heater using acoustic mismatch theory [16].

At sufficiently low temperatures, the phonons travel ballistically through the GaAs and are incident on the 2DES with an angular resolution determined by geometric considerations and phonon focusing. A small proportion of the incident phonon flux is absorbed by the 2DES. This absorbed power causes a rise in the temperature of the 2DES which can be measured with a time resolution of around 50 ns. In the experiment, the temperature increase is deduced from the change in the 2DES resistance which is calibrated against temperature under equilibrium conditions. Care is taken to ensure that the measurement current and repetition rate of the voltage pulse to the heater do not cause spurious heating effects. Due to the small signals and high bandwidth of the measurement system, the signal is averaged over 10^3–10^6 traces to obtain acceptable signal-to-noise ratios.

In figure 2, the typical response of the 2DES (sample B) at a filling factor $\nu = 1/3$ to a pulse of non-equilibrium phonons from the heater directly beneath the meander is shown. Initially the 2DES is at a temperature of 50 mK. After 400 ns, LA phonons travelling directly across the wafer have traversed the 2 mm thick GaAs substrate and hit the 2DES. This leads to only a very small increase in its temperature, showing that the interaction with these LA phonons only plays a minor role in the phonon absorption at this angle of incidence. After 600 ns, τ_{TA}, the temperature of the 2DES, rises much more rapidly. This coincides with the time at which TA phonons travelling almost perpendicular to the interface hit the 2DES. The electronic rise time is fast enough to record the true rate of change of electron temperature with time. The rise time is consistent with the length of the phonon pulse and the angular spread of the phonons (geometrically the maximum angle is $\sim 35°$). Subsequently, with no hot ballistic phonons present in the vicinity of the 2DES, the system starts to cool. At a time corresponding to $3\tau_{TA} = 1.8\ \mu$s further heating is observed. This confirms the interpretation of the heating

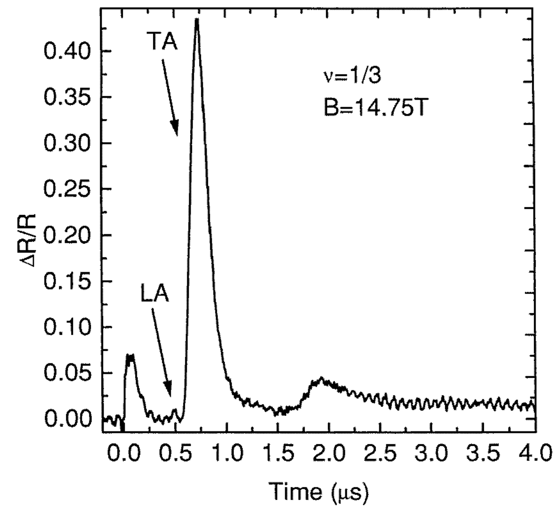

Figure 2. The ballistic response of a 2DES (sample B) at $\nu = 1/3$ to a 20 ns, $T_h = 2.4$ K pulse of phonons. At the start of the pulse the system was at 50 mK. At $t = 0$, a signal due to electrical breakthrough can be seen, followed by a small signal due to the arrival of LA phonons after 0.4 μs. After 0.6 μs the TA phonons arrive; the second rise at $t \simeq 1.8\ \mu$s is the effect of a reflected phonon pulse, demonstrating that the phonon propagation is ballistic. The oscillations at longer times are due to the signal-averaging electronics.

as being due to TA phonons and demonstrates ballistic propagation of the phonons over such long distances. Only after several microseconds do the hot phonons thermalize via inelastic scattering processes inside the substrate leading to a rise in the substrate temperature, T_b.

We will now focus on the results of an experiment on sample A, at $\nu = 1/3$, that is time resolved, but in which the 2DES can absorb phonons over a wide range of angles. It is expected that under these conditions the phonon absorption will be dominated by absorption at the magnetoroton minimum. The frequency spectrum of the ballistic phonons can be altered by changing the heater temperature, T_h. Therefore, in analogy with optical spectroscopy, we can observe the energy absorbed as the 'colour' of the phonon spectrum is altered. Although the resulting phonon spectrum is Planckian, if the absorption line is narrow, useful spectral information can still be obtained.

As the excited magnetorotons at $\nu = 1/3$ are neutral excitations they cannot contribute directly to an increase in resistance. However, provided that they equilibrate with the charged excitations inside the 2DES quickly enough, the energy absorbed will be manifest as an increase in the temperature of the 2DES. To test this hypothesis, we can measure the dependence of the phonon absorption process on the duration of the phonon pulse. The results are shown in figure 3. The total relative energy absorbed, defined as the ratio of the absorbed energy to the energy in the non-equilibrium phonon pulse, is constant. In other words, the total energy absorbed by the 2DES is always proportional to the total energy emitted by the heater, with no saturation observed even at pulse lengths as long as 300 ns. We conclude that the excited magnetorotons do not come into a dynamic equilibrium with the ballistic phonons by emitting phonons of the same energy and wavevectors on the timescales mentioned above. They must therefore equilibrate by heating the 2DES internally without phonon emission into the substrate.

When the phonon pulse reaches the 2DES, a small proportion of the incident phonon energy, $\mathrm{d}E_{ph} = r(T_h)P_h\,\mathrm{d}t$ is absorbed in a time interval $\mathrm{d}t$. Here, P_h is the power dissipated

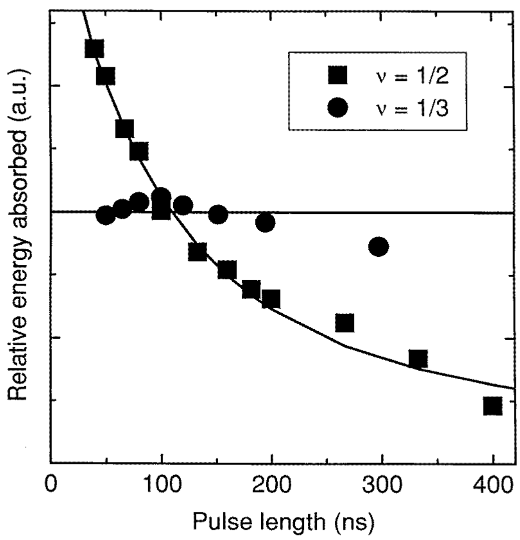

Figure 3. The proportion of the phonon energy absorbed by the 2DES (sample A) at $\nu = 1/3$ and $\nu = 1/2$ as a function of the length of the voltage pulse applied to the heater. The lines represent the behaviour for a system coming into dynamic equilibrium by emitting phonons on timescales of $\tau = 70$ ns ($\nu = 1/2$) and $\tau = \infty$ ($\nu = 1/3$), i.e. equilibrium is not attained.

by the heater in the form of ballistic phonons and $r(T_h)$ is the fraction of phonons absorbed by the 2DES when the non-equilibrium phonon spectrum has a temperature T_h. The energy absorbed by the 2DES will manifest itself as an increase in the electron temperature from its equilibrium value T_0 to a non-equilibrium temperature T_1 given by

$$\int_{T_0}^{T_1} \frac{C(T_e)}{r(T_h)}\, \mathrm{d}T_e = P_{ph}\tau \tag{1}$$

where $C(T_e)$ is the total specific heat of the 2DES.

By measuring the phonon absorption as a function of substrate temperature, T_0, at a fixed heater temperature and pulse length, we can measure the quantity $C(T_e)/r(T_h)$. The relative proportion of phonons absorbed, $r(T_h)$, is expected to be independent of the electron temperature, T_e, for $T_e \ll \Delta_{MR}/k_B$ when the quasiparticle ground state of the 2DES is nearly full and the excited states are almost empty. Therefore, for the temperature range considered, we can regard $r(T_h)$ as independent of T_e (within an accuracy of $\exp(-\Delta_{MR}/k_B T_e)$). It is found that $C(T_e)/rT_e$ is approximately constant below 400 mK [17]. It is expected that the heat capacity for such a system would have a Schottky form for a 2DES in the absence of disorder [18]. As the electron temperatures involved are much less than the magnetoroton energy, the heat capacity due to these excitations will be exponentially small. It is probable therefore that we measure a heat capacity at $\nu = 1/3$ that is due to other degrees of freedom such as localized states and sample inhomogeneity.

Knowing the temperature dependence of the specific heat of the 2DES we can now evaluate the energy absorbed from the ballistic phonons as a function of the temperature of the non-equilibrium phonons. We regard this absorbed energy as an indirect consequence of the excitation of the 2DES across the magnetoroton gap. The fast (<20 ns) decay of the excitations inside the 2DES, i.e. without emission of high-energy phonons, eventually heats the 2DES to a temperature T_1. The total energy absorbed, $E_{abs}(T_h)$, arbitrarily normalized to its value E_0 for $T_h = 1.95$ K and $t = 100$ ns, increases as $(\exp(-\Delta_{MR}/k_B T_e) - 1)^{-1}$ (figure 4). We

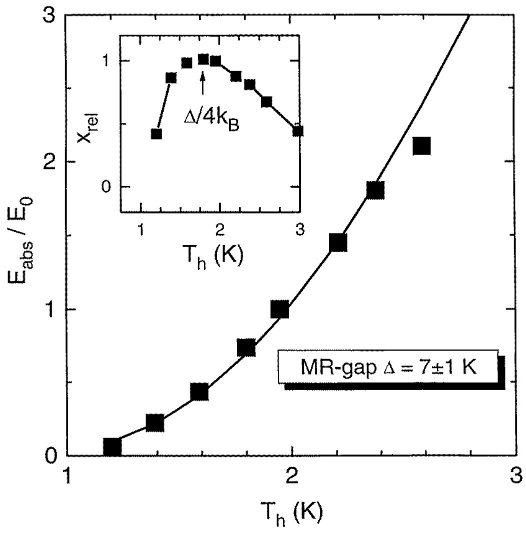

Figure 4. The dependence of the energy absorbed $E_{abs}(T_h)/E_0$ as a function of the heater temperature T_h. The solid curve is the fitted behaviour expected for excitation across a magnetoroton energy gap Δ_{MR}. The inset shows the relative amount of the total phonon energy, x_{rel}, that is absorbed as a function of T_h.

find a magnetoroton energy gap of 7 ± 1 K, twice as high as that determined from transport experiments on the same sample. The energy gap measured by transport experiments is expected to be the larger energy gap at infinite wavevector rather than that at the magnetoroton minimum. Experimentally, it has been found that this measure of the energy gap is reduced by the presence of disorder. The phonon absorption results suggest that the energy required to create a magnetoroton from the strongly correlated Laughlin liquid is largely independent of the disorder in the sample. As the characteristic length scale involved in this process is the magnetic length it might be expected that the process is only sensitive to disorder that varies the properties of the 2DES on length scales shorter than this.

Another method of showing the existence of the energy gap is to plot the ratio, x_{rel}, of the energy absorbed by the 2DES to the energy in the phonon pulse E_{abs}/E_{ph} as a function of heater temperature (see the inset of figure 4). For deformation potential coupling it would be expected that this curve peaks at $T_p \sim \Delta_{MR}/4k_B T_h$ (suggesting $\Delta_{MR} = 7.2 \pm 0.5$ K) where the dominant phonon energy in the black-body spectrum coincides with the magnetoroton gap. The two experimentally determined values for the magnetoroton energy at $\nu = 1/3$ are found to be the same to within experimental error.

For an infinitely thin 2DES, Girvin, MacDonald and Platzman [6] predict that the magnetoroton minimum at $\nu = 1/3$ will be at an energy of $c_{1/3}e^2/4\pi\varepsilon_0\varepsilon l_B$ where $c_{1/3} = 0.075$. Once we have determined the thickness of the 2DES from phonon measurements at $\nu = 1/2$ [17], knowing the magnetic length at $\nu = 1/3$ in sample A, the prediction for $c_{1/3}$ can be corrected for the effects of finite thickness [19]. The predicted value becomes $c_{1/3} = 0.042 \pm 0.002$. Expressing the experimental value in the reduced units, we determine that $c_{1/3} = 0.036 \pm 0.006$. The agreement between the theoretical and experimental values supports the hypothesis that the ballistic phonon measurements are probing the magnetoroton gap at finite in-plane wavevector.

The experiments presented here are good evidence that the acoustic phonons are being absorbed at the magnetoroton minimum. Current research is focusing on angle-resolved measurements and on clarifying the nature of the electron–phonon interaction in the FQH regime. The angle-resolved measurements allow us to place bounds on the range of in-plane wavevectors incident on the 2DES and hold out the possibility of mapping at least part of the magnetoroton dispersion curve [15]. Experiments at other filling factors such as $\nu = 2/5, 2/3$ give qualitatively similar data but smaller energy gaps. Results at non-quantized filling factors produce completely different qualitative behaviour confirming that the results at quantized filling factors are due to the special quantum fluid formed at these filling factors.

3. Magnetically induced Wigner solidification

Experimentally, it is found that the sequence of fractional quantum Hall filling factors does not continue indefinitely. In high-mobility samples at temperatures below 100 mK it is always observed that the sample enters an insulating state at sufficiently high magnetic fields. In gallium arsenide, for a two-dimensional electron system (2DES) it is found that the insulating state occurs at filling factors smaller than $\nu \lessapprox 1/5$ [20] whilst for hole systems (2DHS) the insulating state occurs for $\nu \lessapprox 1/3$ [21]. The nature of this insulating state is intriguing. Is it due to single-particle localization induced by the magnetic field or is this insulating state related to the formation of a 2D Wigner solid which is pinned by the disorder in the system?

Wigner crystals have been observed in several two-dimensional systems such as electrons trapped above the surface of superfluid helium and helium ions trapped underneath the surface. Other authors in this Special Issue describe research on these systems in depth. The first convincing evidence of an experimentally realized electron crystal was found in experiments on electrons trapped at the surface of liquid helium by Grimes and Adams [22] using RF techniques. They found a series of resonances that were identified with the predicted dispersion of phonons in the Wigner crystal (WC). The areal density of carriers in this system, at 10^7 cm^{-2}, is sufficiently low that the system may be regarded as classical. With the higher carrier densities obtainable in semiconductor heterojunctions it was hoped that one could study the formation of the WC in a quantum system. The parameter that determines whether the system crystallizes or not is the ratio of the correlation energy, V_c, to the kinetic energy, K. Wigner predicted that when $V_c \gg K$ a 3D system of dilute carriers can be expected to undergo a transition to a solid-like phase [23]. For this 2D quantum system, the ratio V_c/K can be identified with the ratio of the Wigner–Seitz to the Bohr radius $r_s = a/a_B$ where $a = (\pi n_s)^{-1/2}$ and $a_B = \hbar^2 \varepsilon_0 \varepsilon_r/(m^* e^2)$; m^* is the carrier effective mass, ε_r is the relative dielectric constant for the medium (13 for GaAs) and n_s is the areal density of electrons (typically 10^{10}–10^{11} cm^{-2} in GaAs). For $r_s \gtrapprox 33$ numerical simulations predict the formation of a WC [24]; however, for typical GaAs samples with a carrier density of 4×10^{10} cm^{-2}, $r_s \simeq 3$ for 2DES and $r_s \simeq 15$ for 2DHS and no WC formation is expected.

The likelihood of forming a WC is considerably increased by applying a strong perpendicular magnetic field to quench the kinetic energy of the system, suppressing zero-point-motion effects. The magnetic field allows the carriers to be localized whilst maintaining the minimum possible kinetic energy in the system, and in the high-field limit the system can be described as a system of classical point-like particles and the classical limit is regained. Numerical calculations suggest that in the absence of disorder a 2DES should crystallize at approximately $\nu \lessapprox 1/5$, whilst for a 2DHS, due a larger carrier effective mass, the transition is expected for $\nu \lessapprox 1/3$ [25].

Early experiments by Andrei *et al* [26] used RF absorption to study the insulating states in a high-mobility 2DES. RF resonances were observed in the insulating regime and identified as due to the magnetophonon modes of the magnetically induced Wigner solid. The role of disorder in this system was not fully appreciated until later [27]. It was found that the magnetophonon dispersion relation at wavevectors much smaller than $1/a$ is significantly altered by the presence of disorder which acts to pin the solid and destroy the crystalline order. This leads to the name magnetically induced Wigner solid (MIWS), rather than crystal, for this type of insulating phase.

More recently, the MIWS has been studied in 2DHS using RF techniques at essentially zero [28] and finite wavevector [29]. The latter results were obtained in a collaboration between F I B Williams and co-workers at CEA-Saclay and researchers in Nottingham. These studies rely on a sensitive GHz heterodyne spectrometer developed by the collaboration. The sample used in these studies is a 2DHS with a sheet density of 6×10^{10} cm^{-2} and a low-temperature mobility greater than 8×10^5 cm^2 V^{-1} s^{-1}. The frequency of the microwaves was continuously variable from 0.2 to 4 GHz. The microwave radiation was coupled to the 2DHS by placing it at approximately 200 nm from an open meandering stripline which carries the microwaves from the source to the detector. The fundamental period of the imposed longitudinal electric field is determined by the meander geometry to be 16 μm. This imposed a spatially periodic RF field on the 2DHS allowing magnetophonon modes of several different wavevectors to be studied with this one meander line. A small low-frequency voltage is applied to a gate on the rear of the sample. This resulting electric field modulates the carrier density and allows the use of lock-in techniques to detect any changes in the electric susceptibility of the system as the microwave frequency is varied. This technique differs from a surface acoustic wave experiment in that frequency can be varied at fixed wavevector whereas in a SAW experiment [30] the wavevector varies with frequency. The microwave power in the transmission line is kept to below -70 dBm to prevent heating of the system. Typical absorption resonances have an amplitude of a few parts in 10^3. As can be seen in figure 5, changes in the dispersion and absorption can be easily resolved.

The absorption lines become stronger and narrower as the filling factor is reduced below 0.3. The resonant features also move to higher frequency as the magnetic field is increased (figure 6). The temperature strongly influences the amplitude and width of the absorption lines but has little effect on the frequency of the resonances.

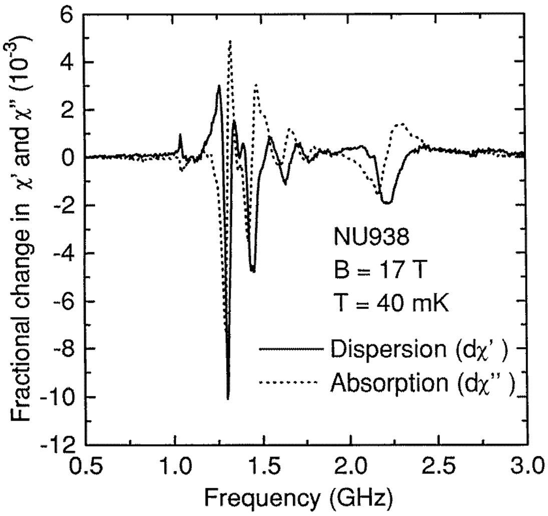

Figure 5. The differential dispersion and absorption as functions of frequency at $B = 17$ T. The power on the microwave transmission line was approximately -70 dBm and the modulation voltage applied to the back-gate was $3V_{RMS}$ at 80 Hz.

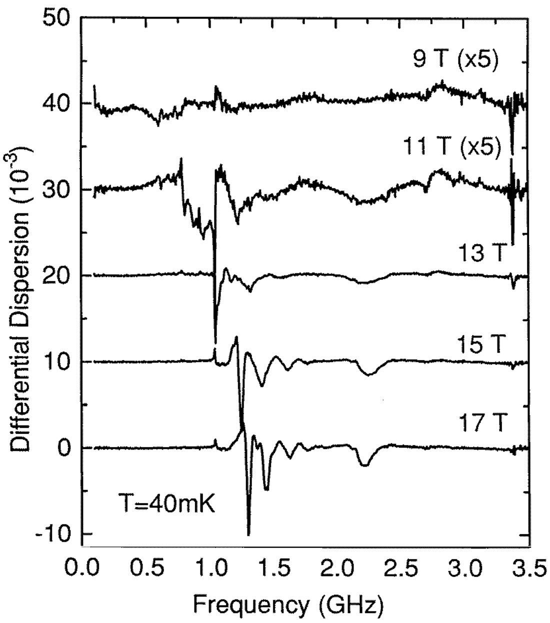

Figure 6. The differential dispersion versus frequency as a function of magnetic field. The absorptions at 1 GHz and 3.6 GHz are due to strong reflections on the microwave transmission line.

The most striking thing about the resonances at high field and low temperature is the small width of the lines. The Q-value of the lowest frequency line exceeds 30 and still has a strong temperature dependence at the lowest temperatures that have been used (40 mK). The reason that this is so surprising is that the disorder is expected to reduce the long-range order in the system and so a broad absorption line might have been expected.

Before detailed comparison can be made to the theory, it is necessary to understand the origin of the resonances. For $\nu < 0.2$ ($B > 12.5$ T), four distinct absorption lines are observed. As the field is reduced, in the region, $0.2 < \nu < 0.3$, the resonances broaden significantly and it is harder to identify the weaker resonances. The effect of the disorder is believed to be to introduce a characteristic frequency, known as the pinning frequency, ω_0, into the problem. In a semiclassical picture, the pinning frequency can be thought of as the natural frequency of shear oscillation of a domain of the MIWS. The centre frequencies, ω, of the absorption lines can be expressed in terms of the plasma frequency, ω_p, the transverse phonon frequency, ω_t, the pinning frequency, ω_0, and the cyclotron frequency, ω_c [27]:

$$\omega(q) = \frac{(\omega_0^2 + \omega_p^2(q))^{1/2}(\omega_0^2 + \omega_t^2(q))^{1/2}}{\omega_c}. \tag{2}$$

At the low wavevectors imposed by the meander line, the transverse frequencies can be neglected in comparison with the pinning frequency. At zero wavevector it is easy to show that $\omega(0) = \omega_0^2/\omega_c$. Using the four resonant lines seen in the experiment, the pinning and plasmon frequencies can be found. It is expected that the plasmon frequencies will be independent of magnetic field and comparable to the fully screened plasmon frequencies, whilst the pinning frequency will vary as the effect of the pinning increases with the magnetic field. Two scenarios can be investigated. The first scenario is that in which the lowest frequency resonance is $\omega(0)$, whilst the second is that in which the lowest-frequency absorption is associated with the fundamental mode of the meander line. The two scenarios are an equally good fit to the data; however, comparing the plasmon frequencies to the fully screened plasma dispersion relation and the fact that the width of the lowest-frequency mode has a stronger temperature dependence than the other absorption lines suggest that the lowest-frequency mode is $\omega(0)$.

Figure 7 shows the field dependences of the plasma and pinning frequencies at 40 mK that result from this analysis. An interesting feature is that there is a reduction in the pinning frequency and an increase in the plasma frequencies at filling factors around $\nu = 1/5$. The reason for this behaviour is not fully understood; however, numerical studies suggest that at a fractional quantum Hall filling factor the quantum state of the system will be an admixture of states with the symmetry of the FQHE with that of the MIWS [31]. This might be expected to reduce the role of pinning in the system, so reducing the pinning frequency. The incompressible nature of the FQHE states may be responsible for the increase in the plasma frequencies.

Ignoring the effects around $\nu = 1/5$, it can be seen that the pinning frequency increases in a slightly sub-linear manner with the magnetic field and the absorption lines become sharper and stronger. Experiments by Li *et al* on a lower-mobility sample of similar density [28] find one broad zero-wavevector resonance that has a very similar frequency dependence to the lowest-frequency resonance that we observe. They find that the oscillator strength of this absorption increases and then becomes constant as the magnetic field is increased, in contradiction to early theoretical predictions [32] which suggest that the oscillator strength will decrease as the field increases. The results of Li *et al* are consistent with those presented above with the exception of the Q-value of the resonance and the behaviour around $\nu = 1/5$. No anomalous behaviour is seen for their lower-mobility sample at this filling factor. The Q-value of the absorption line seen by Li *et al* increases linearly with field to a maximum value of 5 at 14.5 T, whereas our lowest-frequency resonance also increases in a linear fashion but has a Q-value of around 20

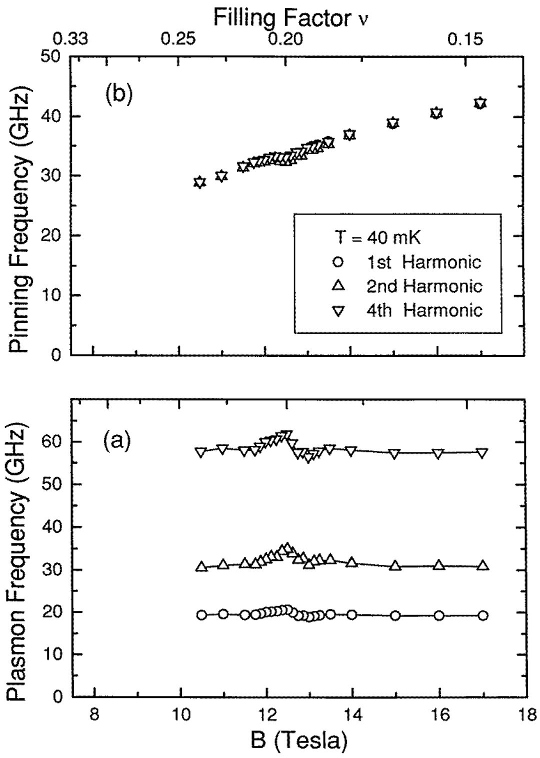

Figure 7. Pinning and plasma frequencies as functions of magnetic field taking the lowest-frequency absorption to be the zero-wavevector mode. As discussed in the text, at around $\nu = 1/5$ a slight dip in the pinning frequency and increase in the plasma frequencies can be seen.

at 14 T and is very sensitive to changes of temperature.

Theoretical work by Chitra *et al* [33] provides the first theoretical scenario in which the pinning frequency increases with magnetic field. The predicted pinning frequency is also of the correct order. However, experimentally the absorptions become stronger and narrower as the field is increased whereas the theoretical model predicts a decreasing absorption strength and increasing linewidth as the magnetic field increases. There is clearly a need for further theoretical and experimental work to clarify the situation.

The microwave absorption experiments have stimulated new theoretical attempts to understand the nature of the insulating phase. The sharp absorption lines seen in both electron and hole systems, the temperature and magnetic field dependences of these absorption lines and their associated linewidths are clear experimental measurements against which theories of the MIWS can be tested. Experimental efforts are now being focused on shorter-wavelength measurements from which it should be possible to measure the shear behaviour of this quantum solid.

4. Conclusions

The finite-wavevector techniques discussed here are providing unique information on the excitations of the quantum liquid and solid states of 2DCS. They also point towards future exciting possibilities such as mapping the dispersion curve of the magnetorotons in the FQHE and the mechanism of the phonon–magnetoroton coupling. The starting point for the theoretical description of the collective excitations was Feynman's description of superfluid helium although, at present, the theory of composite fermions is enabling more detailed calculations of the dispersion curves to be made. Microwave experiments on the insulating phase are showing that even in the presence of disorder the MIWS can still support well-defined excitations which have a high Q-value. In the 2DHS studied, the interplay between the FQHE and MIWS at $\nu = 1/5$ appears to be important. Future experiments are planned at higher wavevectors which should allow the determination of the shear modulus of the MIWS and detailed comparison with theoretical descriptions.

Acknowledgments

The experimental work described would not have been possible without the invaluable efforts of research students R H Eyles, J E Digby, A M Devitt, P F Hennigan and A Beya; postdoctoral researchers U Zeitler, S H Roshko and R Gaal; my colleagues A J Kent, K A Benedict and L J Challis; crystal growers T Cheng and M Henini; F I B Williams in Saclay; and of course Professor W F Vinen who inspired my interest in this field. The research was sponsored by the EPSRC and the EU.

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